1) Velocity field of a flow is given by \( \mathbf{V} = ai + bxj \), where \( a = 3 \text{ m/s} \) and \( b = 2 \text{ s}^{-1} \). Coordinates are measured in meters.
   
a) Obtain the equation for the streamline passing through point (2, 4).
   
b) At \( t = 5 \text{ s} \), what are the coordinates of the particle that passed through point (0, 4) at time \( t = 0 \text{ s} \)?
   
c) What conclusion can you draw about the pathline, streamline and streakline for this flow?
2) Crude oil, with specific gravity \( \text{SG}=0.85 \) and viscosity \( \mu=0.1 \text{ Ns/m}^2 \), flows steadily down a surface inclined \( \theta=45^\circ \) in a film of thickness \( h=2.5 \text{ mm} \). The velocity profile is given by the expression below. (Coordinates \( x \) is along the surface and \( y \) is normal to the surface.) Plot the velocity profile. Determine the magnitude and direction of the shear stress that acts on the surface.

\[
    u = \frac{\rho g}{\mu} \left( h y - \frac{y^2}{2} \right) \sin \theta
\]

**Given:** Velocity profile.

**Find:** Plot of velocity profile; shear stress on surface.

**Solution:**

The velocity profile is

\[
    u = \frac{\rho g}{\mu} \left( h y - \frac{y^2}{2} \right) \sin \theta
\]

so the maximum velocity is at \( y = h \)

\[
    u_{\text{max}} = \frac{\rho g}{\mu} \cdot \frac{h^2}{2} \cdot \sin (\theta)
\]

Hence we can plot

\[
    \frac{u}{u_{\text{max}}} = 2 \left[ \frac{y}{h} - \frac{1}{2} \left( \frac{y}{h} \right)^2 \right]
\]

![Graph of velocity profile](image)

This graph can be plotted in Excel.

The given data is:
- \( h = 2.5 \text{ mm} \)
- \( \mu = 0.1 \text{ Ns/m}^2 \)
- \( \theta = 45^\circ \text{ deg} \)

Basic equation

\[
    \tau_{yx} = \mu \cdot \frac{du}{dy}
\]

\[
    \tau_{yx} = \mu \cdot \frac{du}{dy} = \mu \cdot \frac{d}{dy} \left( \frac{\rho g}{\mu} \left( h y - \frac{y^2}{2} \right) \sin (\theta) \right) = \rho g \cdot (h - y) \cdot \sin (\theta)
\]

At the surface \( y = 0 \)

\[
    \tau_{yx} = \rho g \cdot h \cdot \sin (\theta)
\]

Hence

\[
    \tau_{yx} = 0.85 \times 1000 \frac{\text{kg}}{\text{m}^3} \times 9.81 \text{ m/s}^2 \times 2.5 \text{ mm} \times \frac{1 \text{ m}}{1000 \text{ mm}} \times \sin (45^\circ) \times \frac{\text{N} \text{s}^2}{\text{kg} \cdot \text{m}} \times \tau_{yx} = 14.74 \frac{\text{N}}{\text{m}^2}
\]

The surface is a positive \( y \) surface. Since \( \tau_{yx} > 0 \), the shear stress on the surface must act in the plus \( x \) direction.
3) A block weighing 45 N and having dimensions 250 mm on each edge is pulled up on an inclined surface on which there is a film of SAE 10W oil at 37°C. If the speed of the block is 0.6 m/s and the oil film is 0.025 mm thick, find the force required to pull the block. Assume that velocity profile in the oil film is linear. The surface is inclined at an angle of 25° from the horizontal.

Given: Block pulled up incline on oil layer.

Find: Force required to pull the block

Solution:

Governing equations: $\tau_{yx} = \mu \frac{du}{dy}$

$\sum F_x = M \cdot a_x$

Assumptions: Laminar flow

The given data is  
- $W = 45$ N  
- $U = 0.6 \frac{m}{s}$  
- $w = 250$ mm  
- $d = 0.025$ mm  
- $\theta = 25^\circ$ deg

$\mu = 3.7 \times 10^{-2} \frac{N \cdot s}{m^2}$  
Fig. A.2 @ 37°C

Equation of motion $\sum F_x = M \cdot a_x = 0$  
so  
$F - f - W \cdot \sin(\theta) = 0$

The friction force is $f = \tau_{yx} \cdot A = \mu \frac{du}{dy} \cdot A = \mu \frac{U}{d} \cdot w^2$

Hence  
$F = f + W \cdot \sin(\theta) = \mu \frac{U}{d} \cdot w^2 + W \cdot \sin(\theta)$

$F = 3.7 \times 10^{-2} \frac{N \cdot s}{m^2} \times 0.6 \frac{m}{s} \times \frac{1}{0.025 \text{ mm}} \times \frac{1000 \text{ mm}}{m} \times \left( \frac{250 \text{ mm} \times \frac{m}{1000 \text{ mm}}}{1000 \text{ mm}} \right)^2 + 45 \text{ N} \sin(25 \text{ deg})$

$F = 74.52 \text{ N}$
4) A concentric cylinder viscometer is driven by a falling mass $M = 0.20 \text{ kg}$ connected by a cord and pulley to the inner cylinder, as shown. The liquid to be tested fills the annular gap of width $a = 0.4\text{mm}$ and height $H = 160 \text{ mm}$. After a brief starting transient, the mass falls at constant speed $V_m = 60 \text{ mm/s}$. Develop an algebraic expression for the viscosity of the liquid in the device in terms of $M$, $g$, $V_m$, $r$, $R$, $a$, and $H$. Evaluate the viscosity of the liquid. Note: $r = 50 \text{ mm}$, $R = 100 \text{ mm}$.

**Solution:**

Apply Newton's law of viscosity

**Basic Equation:**

$$
\tau - \mu \frac{du}{dy} = 0;
\sum M = 0;
T = \tau AR
$$

**Assumption:**

i) Newtonian liquid  
ii) Narrow gap, so linear profile  
iii) Steady angular speed

Summing torque on the rotor

$$
\sum M = Mg r - \tau AR = 0; \ A = 2\pi RH
$$

Because $a \ll R$, treat the gap as plane. Then

$$
\tau = \frac{du}{dy} = \mu \frac{U}{a} = \mu \left( \frac{V}{a} \right) = \mu \frac{Vr}{ar}
$$
On substitution

\[ \frac{Mg r}{\left( \frac{\mu V_m}{r} \right)} (2\pi RHR) = 0 \]

\[ Mgr - \left( \frac{2\pi \mu V_m R^3 H}{ar} \right) = 0 \]

So,

\[ \mu = \frac{Mgr a}{2\pi V_m R^3 H} \] (1)

Substitute corresponding values in equation (1)

\[ \mu = \frac{1}{2\pi} (0.2 \text{ kg}) (9.81 \text{ m/s}^2) (0.050)^3 \text{ m}^2 (0.0004) \left( \frac{5}{0.060 \text{ m}} \right) \]

\[ \times \frac{1}{(0.1)^3 \text{ m}^3} \times \frac{1}{0.160 \text{ m}} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \]

\[ = 0.0325 \text{ N} \cdot \text{s/m} \]

\[ = 32.5 \text{ mPa} \cdot \text{s} \]

Hence, the viscosity of the liquid is \[ 32.5 \text{ mPa} \cdot \text{s} \].