FUNDAMENTAL CONCEPTS FOR FLOW ANALYSIS

We covered methods of analysis of nonflowing fluids in the previous chapter. In this chapter, we develop the fundamental concepts of flow analysis, including the way to describe fluid flow, natural laws that govern fluid flow, different approaches to formulating mathematical models of fluid flow, and methods that engineers use to solve flow problems.

The Fundamental Laws

Experience have shown that all fluid motion analysis must be consistent with the following fundamental laws of **nature.**

The law of conservation of mass: Mass can be neither created nor destroyed. It can only be transported or stored.

Newton's three laws of motion:
A mass remains in a state of equilibrium, that is, at rest or moving at constant velocity, unless acted on by an unbalanced force.

The rate of change of linear momentum of mass is equal to the net force acting on the mass.

Any force action has a force reaction equal in magnitude and opposite in direction.

The first law of thermodynamics (law of conservation of energy) Energy, like mass, can be neither created nor destroyed. Energy can be transported, changed in form, or stored.

The second law of thermodynamics: The entropy of the universe must increase or, in the ideal case, remain constant in all natural processes.

The state of postulate (law of property relations): The various properties of a fluid are related. If a certain minimum number (usually two) of fluid's properties are specified, the remainder of the properties can be determined.

NOTE: **These laws apply to all flows.** They do not depend on the nature of the fluid, the geometry of the boundaries, or anything else. As far as we know, they have always been true and will continue to be true unless they are suspended by the creator of the universe. **Hence, we can firmly base analysis of all flows on these laws.**

Constitutive Relations

In addition to these universal laws, several less fundamental laws, such as Newton's law of viscosity, Fourier's law of conduction, are needed to solve flow problems. **These laws are true for some fluids.**

Mathematical Formulation

The fundamental laws are the basis of our understanding of fluid motion. However, besides understanding, an engineer needs to know quantitatively the velocity, and the pressure to calculate the effects of the fluid on surfaces that it contacts, such as force exerted by the fluid on a surface, pressure drop in a pipe flow, etc.

To obtain predictive capability, the fundamental laws must be expressed mathematically and they must be solved to predict velocity or pressure.

To formulate the fundamental laws, we choose both a point of view and a mathematical method.

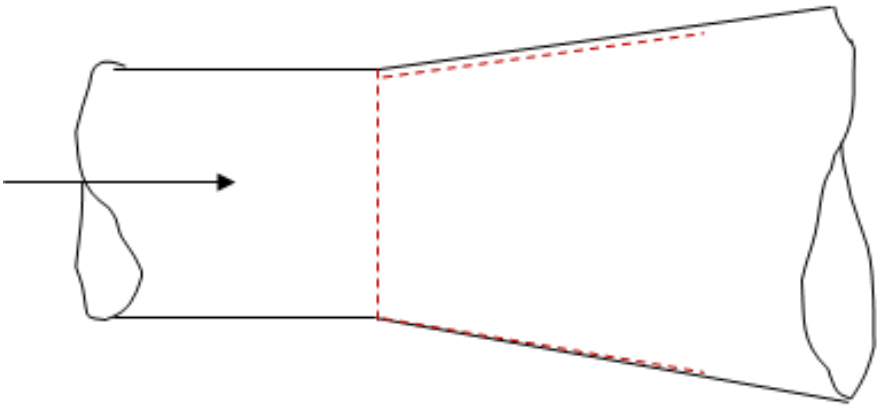
- **System versus Control Volume (Lagrangian vs Eulerian point of view)**
- **Differential versus integral formulation**

System versus Control Volume

We may apply the fundamental laws to either a system or a control volume.

System : a specific fluid mass selected for analysis.
Control Volume: a specific region of space selected for analysis.

System and control volume may be either infinitesimally small or finite.



The system point of view is related to a Lagrangian description of flow. Its advantages is that all the fundamental laws may be expressed directly in terms of a specific collection of mass.

Control volume point of view is related to an Eulerian description of flow. Its advantage is that control volumes are easier to use for problem solution.

Thus we adopt the system point of view to formulate the fundamental laws, but use the control volume point of view to apply them to problems. Fortunately, we can formally connect the two points view by purely mathematical relationships.

Differential versus Integral Formulation

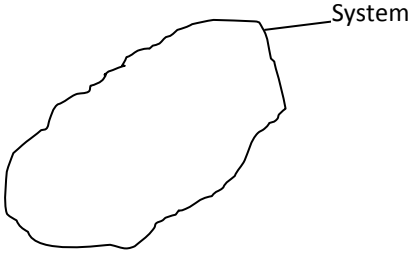
We must now consider the level of detail of the resulting flow analysis. We must choose between a detailed point by point description and a global or lumped description.

When **a point by point (local) description** is desired, fundamental laws are applied to an infinitesimal control volume. **The result will be a set of differential equations with the fluid velocity and pressure as dependent variables and the location (x, y, z) and time as independent variables.** Solution of these differential equations, together with boundary conditions, will be two functions V(x, y, z, t), and P(x, y, z, t) that can tell us the velocity and pressure at every point.

When **global information** such as flow rate, force and temperature change between inlet and outlet is desired, the fundamental laws are applied to a finite control volume. **The result will be a set of integral equations.**

BASIC LAWS FOR A SYSTEM

Conservation of Mass



$$\left.\frac{dM}{dt}\right|_{system} = 0 \quad \text{where } M_{sys} = \int_{\substack{mass\\(sys)}} dm = \int_{\forall\ sys} \rho d\forall$$

$$\frac{d}{dt}\int_{\forall\ sys} \rho d\forall = 0$$

Newton's Second Law

$$\vec{F} = \left.\frac{d\vec{P}}{dt}\right|_{sys} \qquad \vec{P}: \textit{linear momentum} \qquad \vec{P} = \int_{\substack{mass\\(sys)}} \vec{V} dm = \int_{\forall\ sys} \vec{V}\rho d\forall \Rightarrow \vec{F} = \frac{d}{dt}\int_{\forall sys} \vec{V}\rho d\forall$$

The First Law of Thermodynamics

$$\delta Q - \delta W = dE \quad \text{in the rate form} \quad \dot{Q} - \dot{W} = \left.\frac{dE}{dt}\right|_{sys} \Rightarrow \dot{Q} - \dot{W} = \frac{d}{dt}\int_{\forall sys} e\rho d\forall$$

The total energy of the system is given by

$$E_{sys} = \int_{\substack{mass\\(sys)}} e dm = \int_{\forall\ sys} e\rho d\forall \quad where \quad e = u + \frac{v^2}{2} + gz$$

The Second Law of Thermodynamics

If an amount of heat is transferred to a system at temperature T, the second law of thermodynamics states that the change in entropy ***ds*** of the system is given by

$$dS \geq \frac{\delta Q}{T} \quad \text{on the rate basis} \quad \left.\frac{dS}{dt}\right|_{sys} \geq \frac{1}{T}\dot{Q} \quad \Rightarrow \quad \frac{d}{dt}\int_{\forall sys} s\rho d\forall \geq \frac{1}{T}\dot{Q}$$

Total entropy of the system is $S_{sys} = \int_{\substack{mass\\(sys)}} s dm = \int_{\substack{\forall\\(sys)}} \rho s d\forall$

Angular Momentum Principle

$$\vec{T} = \left.\frac{d\vec{H}}{dt}\right|_{sys} \qquad \vec{H}: \textit{angular momentum} \qquad \vec{H} = \int_{\substack{mass\\(sys)}} (\vec{r}\times\vec{V}) dm = \int_{\forall\ sys} (\vec{r}\times\vec{V})\rho d\forall$$

$$\vec{T} = \frac{d}{dt}\int_{\forall\ sys} (\vec{r}\times\vec{V})\rho d\forall$$

RELATION OF SYSTEM DERIVATIVES TO THE CONTROL VOLUME FORMULATION

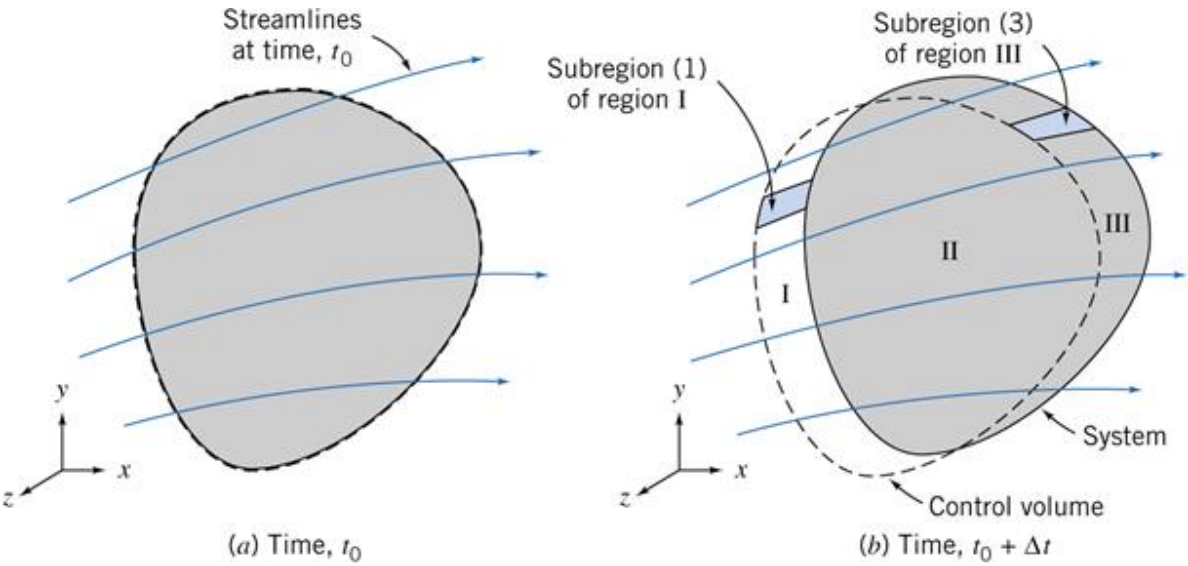
The above equations involve the time derivative of an extensive property of the system (mass, momentum, energy, entropy). All the above equations can be expressed in terms of a general intensive property η. Thus

$$N_{sys} = \int_{\substack{mass \\ (sys)}} \eta dm = \int_{\substack{\forall \\ (sys)}} \eta \rho d\forall$$

Comparing this with the above equations, we see that when

N=M	then	η=1	N=E	then	η=e
N= $\vec{P}$	then	η= $\vec{V}$	N=S	then	η=s

Consider a system and control volume whose **boundaries coincide at t_0**.



Objective: To relate the $\left.\frac{dN}{dt}\right|_{system}$ to the time variations of this property (N) associated with the control volume.

From the definition of a derivative,

$$\left.\frac{dN}{dt}\right|_{system} \equiv \lim_{\Delta t \to 0} \frac{N_s)_{t_0+\Delta t} - N_s)_{t_0}}{\Delta t} \qquad (1)$$

At ***t+Δt***, the system occupies regions II and III, at t_0, the system and the control volume coincide, we can write

$$N_s\big)_{t_0} = \big(N_{C\forall}\big)_{t_0} = \int_{C\forall} \eta\rho d\forall \tag{2}$$

$$N_s\big)_{t_0+\Delta t} = \big(N_{II} + N_{III}\big)_{t_0+\Delta t} = \big(N_{C\forall} - N_I + N_{III}\big)_{t_0+\Delta t}$$

$$N_s\big)_{t_0+\Delta t} = \left[\int_{C\forall} \eta\rho d\forall\right]_{t_0+\Delta t} - \left[\int_{I} \eta\rho d\forall\right]_{t_0+\Delta t} + \left[\int_{III} \eta\rho d\forall\right]_{t_0+\Delta t} \tag{3}$$

Substituting these expressions şnto the definition of the system derivative

$$\left.\frac{dN}{dt}\right)_{system} = \lim_{\Delta t\to 0} \frac{\left[\int_{C\forall} \eta\rho d\forall\right]_{t_0+\Delta t} - \left[\int_{I} \eta\rho d\forall\right]_{t_0+\Delta t} + \left[\int_{III} \eta\rho d\forall\right]_{t_0+\Delta t} - \left[\int_{C\forall} \eta\rho d\forall\right]_{t_0}}{\Delta t}$$

or

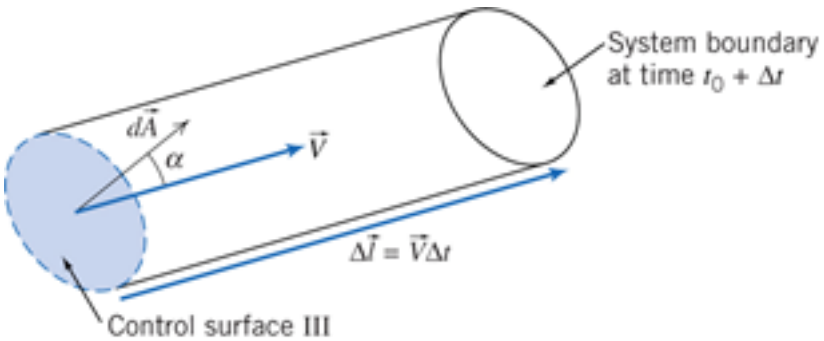
$$\left.\frac{dN}{dt}\right)_{system} = \underbrace{\lim_{\Delta t\to 0} \frac{\left[\int_{C\forall} \eta\rho d\forall\right]_{t_0+\Delta t} - \left[\int_{C\forall} \eta\rho d\forall\right]_{t_0}}{\Delta t}}_{1} + \underbrace{\lim_{\Delta t\to 0} \frac{\left[\int_{III} \eta\rho d\forall\right]_{t_0+\Delta t}}{\Delta t}}_{2} - \underbrace{\lim_{\Delta t\to 0} \frac{\left[\int_{I} \eta\rho d\forall\right]_{t_0+\Delta t}}{\Delta t}}_{3} \tag{4}$$

Term 1 in Eq. 4 simplifies to

$$\lim_{\Delta t\to 0} \frac{\left[\int_{C\forall} \eta\rho d\forall\right]_{t_0+\Delta t} - \left[\int_{C\forall} \eta\rho d\forall\right]_{t_0}}{\Delta t} = \frac{\partial}{\partial t}\int \eta\rho d\forall \tag{5}$$

$$\lim_{\Delta t\to 0} \frac{\left[\int_{III} \eta\rho d\forall\right]_{t_0+\Delta t}}{\Delta t} = \lim_{\Delta t\to 0} \frac{N_{III}\big)_{t_0+\Delta t}}{\Delta t}$$

To evaluate $N_{III})_{to+\Delta t}$, let us look at an enlarged view of a typical subregion of region III.



$$\left.\begin{aligned} N_{III}\big)_{t_0+\Delta t} &= \left[\int_{III} \eta\rho d\forall\right]_{t_0+\Delta t} \\ d\forall &= \Delta l\cos\alpha dA \end{aligned}\right\} \Rightarrow N_{III}\big)_{t_0+\Delta t} = \left[\int_{CS_{III}} \eta\rho\Delta l\cos\alpha dA\right]_{t_0+\Delta t}$$

Note: The angle α will always be less than $\pi/2$ over the entire area of the control surface bounding region III.

In the above expression, Δl is the distance travelled by a particle on the system surface during the time interval Δt along the streamline that existed at t_0.

$$\lim_{\Delta t\to 0}\frac{\left[\int_{III}\eta\rho d\forall\right]_{t_0+\Delta t}}{\Delta t}=\lim_{\Delta t\to 0}\int_{CS_{III}}\eta\rho\frac{\Delta l}{\Delta t}\cos\alpha dA$$

Note: $\lim_{\Delta t\to 0}\frac{\Delta l}{\Delta t}=\left|\vec{V}\right|$ and $dA=\left|d\vec{A}\right|$

Hence

$$\lim_{\Delta t\to 0}\frac{\left[\int_{III}\eta\rho d\forall\right]_{t_0+\Delta t}}{\Delta t}=\int_{CS_{III}}\eta\rho\left|\vec{V}\right|\cos\alpha\left|d\vec{A}\right| \tag{6}$$

The term 3 in Eq. 4 simplifies to

$$-\lim_{\Delta t\to 0}\frac{\left[\int_{I}\eta\rho d\forall\right]_{t_0+\Delta t}}{\Delta t}=\lim_{\Delta t\to 0}\frac{N_I)_{t_0+\Delta t}}{\Delta t}$$

To evaluate $N_I)_{t_0+\Delta t}$, look at an enlarged view of a typical subregion

ŞEKİL VAR

$$d\forall=\Delta l(-\cos\alpha)dA$$

$$-\lim_{\Delta t\to 0}\frac{\left[\int_{I}\eta\rho d\forall\right]_{t_0+\Delta t}}{\Delta t}=-\lim_{\Delta t\to 0}\frac{\int_{CS_I}\eta\rho\Delta l\cos\alpha dA}{\Delta t}$$

$$=+\lim_{\Delta t\to 0}\int_{CS_I}\eta\rho\frac{\Delta l}{\Delta t}\cos\alpha dA=+\int_{CS_I}\eta\rho\left|\vec{V}\right|\cos\alpha\left|d\vec{A}\right| \tag{7}$$

Substituting Eqs. (5),(6) and (7) into (4)

$$\left.\frac{dN}{dt}\right)_{sys} = \frac{\partial}{\partial t}\int_{C\forall}\eta\rho d\forall + \int_{CS_{III}}\eta\rho\left|\vec{V}\right|\cos\alpha\left|d\vec{A}\right| + \int_{CS_I}\eta\rho\left|\vec{V}\right|\cos\alpha\left|d\vec{A}\right|$$

$$CS = CS_I + CS_{III}$$

Hence we can write,

$$\left.\frac{dN}{dt}\right)_{sys} = \frac{\partial}{\partial t}\int_{C\forall}\eta\rho d\forall + \int_{CS}\eta\rho\left|\vec{V}\right|\cos\alpha\left|d\vec{A}\right|$$

Recognizing that $\left|\vec{V}\right|\cos\alpha\left|d\vec{A}\right| = \vec{V}\cdot d\vec{A}$

$$\left.\frac{dN}{dt}\right)_{sys} = \frac{\partial}{\partial t}\int_{C\forall}\eta\rho d\forall + \int_{CS}\eta\rho\vec{V}\cdot d\vec{A}$$

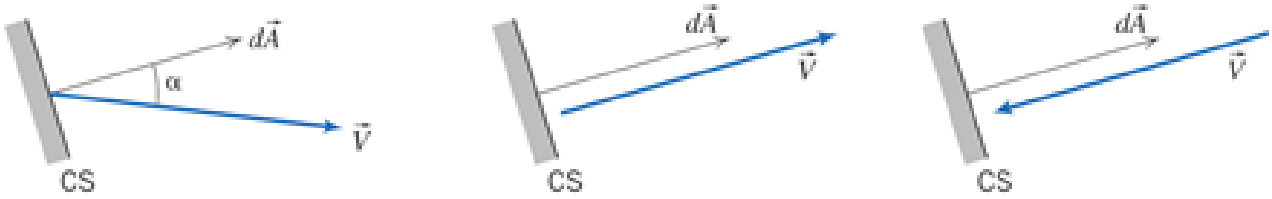
It is important to recall that in deriving the above equation, the limiting process (taking the limit as $\Delta t \to 0$) ensured that the relation is valid at the instant when the system and control volume coincide.

$\left.\frac{dN}{dt}\right)_{sys}$: the total rate of change of any arbitrary extensive property of the system.

$\frac{\partial}{\partial t}\int_{C\forall}\eta\rho d\forall$: the time rate of change of the arbitrary extensive property N within the control volume

$\int_{CS}\eta\rho\vec{V}\cdot d\vec{A}$: the net rate of flux of the extensive property N through the control surface.

Evaluating the scalar product



$\vec{V}\cdot d\vec{A} = VdA\cos\alpha$ (a) General inlet/exit

$\vec{V}\cdot d\vec{A} = +VdA$ (b) Normal exit

$\vec{V}\cdot d\vec{A} = -VdA$ (c) Normal inlet

BASIC EQUATIONS FOR A CONTROL VOLUME

Basic equations for a control volume are obtained by combining the basic equations for a system and Reynolds transport theorem.

EQUATION OF CONSERVATION OF MASS (Continuity Equation)

Combining the law of conservation of mass with the transport theorem yields one of the most useful equations in all fluid mechanics: the continuity equation. Recall that conservation of mass states simply that the mass of a system is constant,

$$\left.\frac{dM}{dt}\right)_{sys} = 0, \qquad M_{sys} = \int_{\forall_{SYS}} \rho d\forall$$

The system and control volume formulation of the conservation of mass are related by Reynolds transport theorem. Setting, **N=M** then η **=1**, we obtain

$$\left.\frac{dM}{dt}\right)_{sys} = \frac{\partial}{\partial t}\int_{C\forall} \rho d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A} = 0$$

$$\frac{\partial}{\partial t}\int_{C\forall} \rho d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A} = 0$$ Continuity equation for a finite control volume

$\frac{\partial}{\partial t}\int_{C\forall} \rho d\forall$ rate of change of mass within the control volume

$\int_{CS} \rho \vec{V}\cdot d\vec{A}$ net rate of flux through the control surface

NOTE: $\vec{V}$ is the velocity measured relative to the control surface. The sign of the dot product $\rho \vec{V}\cdot d\vec{A}$ depends on the direction of velocity vector $\vec{V}$, relative to the area vector $d\vec{A}$. $\rho \vec{V}\cdot d\vec{A}$ is positive where flow is out through the control surface, negative where flow is in through the control surface, and zero where flow is tangent to surface.

Special Cases

1. Incompressible Flow

For incompressible flow, ρ = constant

$$\frac{\partial}{\partial t}\rho\int_{C\forall} d\forall + \rho\int_{CS}\vec{V}\cdot d\vec{A} = 0$$

$$\frac{\partial}{\partial t}[\rho\forall] + \rho\int_{CS}\vec{V}\cdot d\vec{A} = 0$$

$$\int_{CS}\vec{V}\cdot d\vec{A} = 0$$

$$\frac{\partial}{\partial t}[\rho\forall] = 0 \quad \begin{bmatrix}\rho = \text{constant}\\ \forall = \text{constant}\end{bmatrix}$$

$\int \vec{V}\cdot d\vec{A}$ is called the volume flow rate of flow over a section of the control surface.

2. Steady Flow

$$\frac{\partial}{\partial t} = 0$$

Hence the continuity equation becomes,

$$\int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad \text{[flow could be compressible]}$$

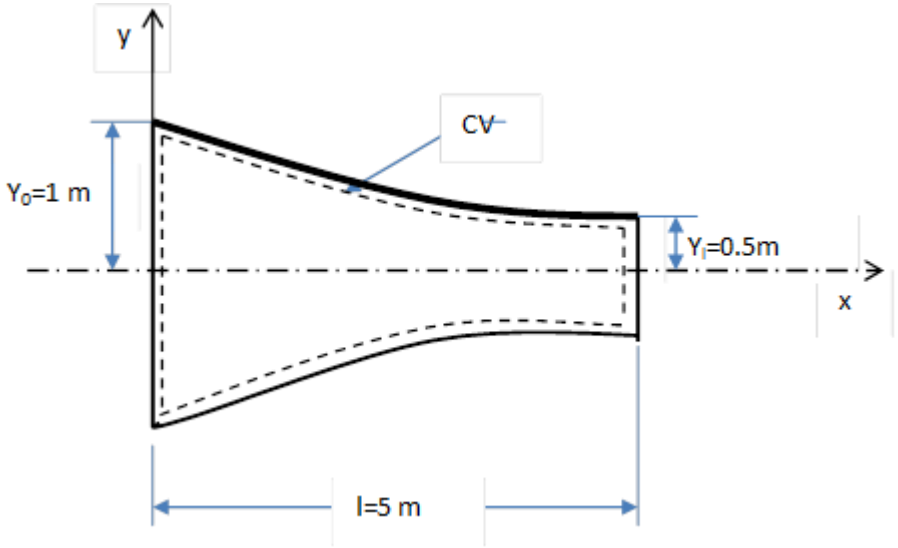
3. Uniform Flow

The velocity is constant across the entire area at a section when density is also constant at a section, then at section **n**

$$\int_{A_n}\rho\vec{V}\cdot d\vec{A} = \rho_n\vec{V}_n\cdot\vec{A}_n = \pm\left|\rho_n V_n A_n\right|$$

Example:

A constant density fluid flows in the converging, two-dimensional channel shown in the figure. The width perpendicular to the paper is quite large compared to the channel height. The velocity in the z-direction is zero. The channel half height Y and the fluid velocity in x-direction are given by



$$Y = \frac{y_0}{1 + x/l}$$

$$u = u_0\left(1 + \frac{x}{l}\right)\left[1 - \left(\frac{y}{Y}\right)^2\right]$$

where u_0=1.0 m/s

Show that the flow field satisfies the continuity equation.

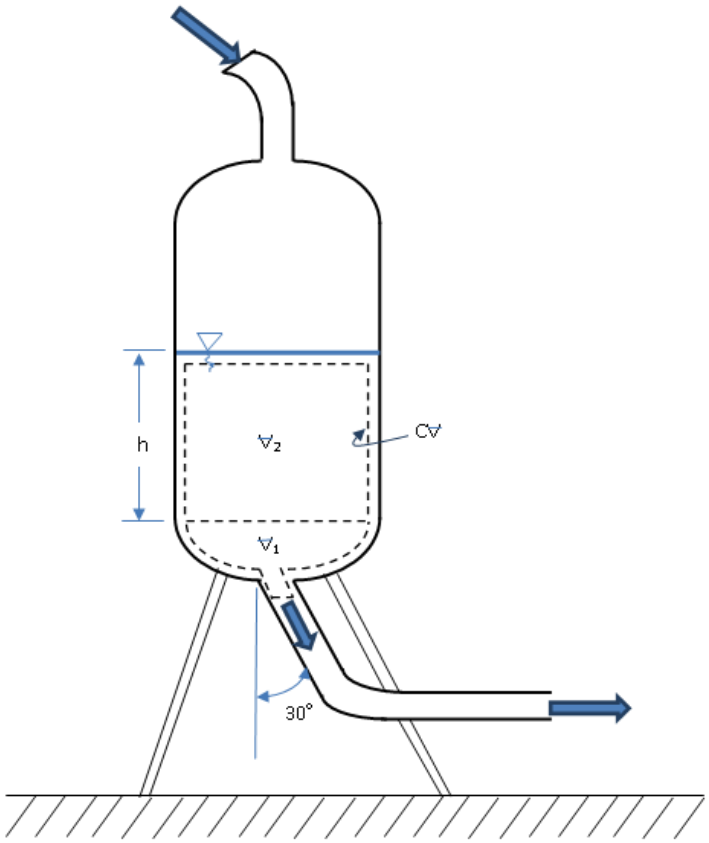
Solution:

To be completed in class

Example:

Water is being added to a storage tank at the rate of **2000 lt/min**. At the same time, water flows through a **5 cm** inside diameter pipe with an average velocity of **18 m/s**. The storage tank has an inside diameter of **300 cm**. Find the rate at which the water level rises or falls.

flow rate 2000 lt/min.
storage tank diameter 300 cm
discharge pipe diameter 5 cm
discharge velocity 18 m/s



Solution:

To be completed in class

MOMENTUM EQUATION FOR AN INERTIAL CONTROL VOLUME

In this section, we will develop mathematical formulation of Newton's Second Law **for an inertial control volume**.

Inertial control volume is the control volume that is not accelerating relative to a stationary frame of reference.

Recall that Newton's second law for a system moving relative to an inertial coordinate system was

$$\vec{F} = \left.\frac{d\vec{P}}{dt}\right)_{sys}$$

$$\vec{P} = \int_{\forall_{sys}} \vec{V}\rho d\forall \quad \text{linear momentum}$$

F = total resultant force

Using the relation between the system and control volume formulations

$$\left.\frac{dN}{dt}\right)_{sys} = \frac{\partial}{\partial t}\int_{C\forall} \eta\rho d\forall + \int_{CS} \eta\rho\vec{V}\cdot d\vec{A}$$

and setting N=**P** and η = **V** , we obtain

$$\left.\frac{d\vec{P}}{dt}\right)_{sys} = \frac{\partial}{\partial t}\int_{C\forall} \vec{V}\rho d\forall + \int_{CS} \vec{V}\rho\vec{V}\cdot d\vec{A}$$

Note: $\left.\frac{d\vec{P}}{dt}\right)_{sys} = \vec{F}\Big)_{on\ sys}$

Since, in deriving the relation between the system and control volume formulation, the system and control volume coincided at t_o

$$\vec{F}\Big)_{on\ sys} = \vec{F}\Big)_{on\ control\ volume}$$

Hence, we can write,

$$\vec{F} = \frac{\partial}{\partial t}\int_{C\forall} \vec{V}\rho d\forall + \int_{CS} \vec{V}\rho\vec{V}\cdot d\vec{A}$$

MOMETUM EQUATION

Total force acting on control volume is equal to summ of surface and body forces, i.e.

$$\vec{F} = \vec{F}_S + \vec{F}_B$$

$$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{C\forall} \vec{V}\rho d\forall + \int_{CS} \vec{V}\rho\vec{V}\cdot d\vec{A}$$ MOMETUM EQUATION

This equation states that the sum of all forces (surface and body forces) acting on a nonaccelerating control volume is equal to the sum of the rate of the change of momentum inside the control volume and the net rate of efflux of momentum through the control surface.

$\vec{F}_S = \int_A -p\, d\vec{A}$ surface force due to pressure

$\vec{F}_B = \int_{C\forall} \rho\, \vec{g}\, d\forall$ body force due to gravity

Sometimes surface force $\mathbf{F_s}$ may also include shear force.

The momentum equation is a vector equation. From this vector equation, a scalar component in each direction can be written, i.e.

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{C\forall} u\rho d\forall + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

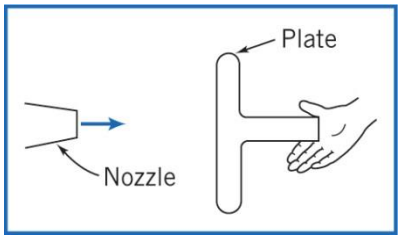
$$F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{C\forall} v\rho d\forall + \int_{CS} v\rho\vec{V}\cdot d\vec{A}$$

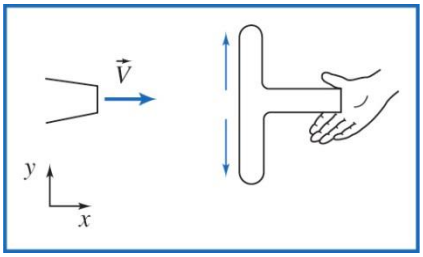
$$F_z = F_{S_z} + F_{B_z} = \frac{\partial}{\partial t}\int_{C\forall} w\rho d\forall + \int_{CS} w\rho\vec{V}\cdot d\vec{A}$$

The momentum equation is usually used to calculate force interactions between a moving fluid and solid objects in contact with it.

Example:

Water from a stationary nozzle strikes a flat plate as shown. The velocity of the water leaving the nozzle is **15 m/sec**. The nozzle exit cross-sectional area is **0.01 m²**. Assuming the water is directed normal to the plate; **determine the horizontal force on the support.**





Horizontal force K_x = ?

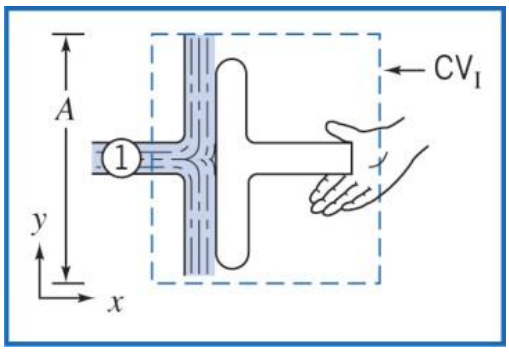
Solution

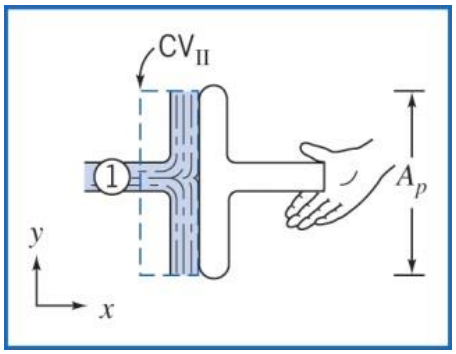
Assumptions:

- Steady flow
- Incompressible flow
- Uniform flow at each section where flow crosses the CV surface

Since the force interaction between the fluid and the solid object is the point of interest, we have to use momentum equation.

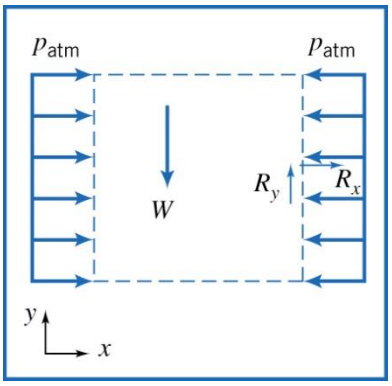
We must choose a suitable control volume. A number of possible choices are,





Note: Regardless of our choice of control volume, the result should be the same.

I. Use C∀$_I$



Momentum equation in x-direction

$$F_x = F_{S_x} + F_{B_x} = \underbrace{\frac{\partial}{\partial t}\int_{C\forall} u\rho d\forall}_{\substack{=0 \\ Steady\ flow}} + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

F_{Bx} = 0, no body force in x-direction

$$F_{S_x} = \underbrace{p_A A}_{\substack{pressure\ force \\ on\ left\ face}} - \underbrace{p_A A}_{\substack{pressure\ force \\ on\ right\ face}} + \underbrace{R_x}_{\substack{force\ of\ the\ support \\ on\ CV\ (assumed\ positive)}}$$

$$F_{S_x} = R_x$$

NOTE: Left and right faces of the control volume are equal.

Rx is the force exerted by the support (hand) on the control volume

$$R_x = \int_{CS} u\rho\vec{V}\cdot d\vec{A} = \int_{A_1} u\rho\vec{V}\cdot d\vec{A}$$

At 1 , $\rho\vec{V}\cdot d\vec{A} = -|\rho V_1 dA|$, since direction of area is 180° apart from the velocity.

$$R_x = \int_{A_1} u\{-|\rho V_1 dA|\}$$

For top and bottom surfaces, u=0

$$R_x = -u_1|\rho V_1 A_1|$$

{properties are uniform over A_1}

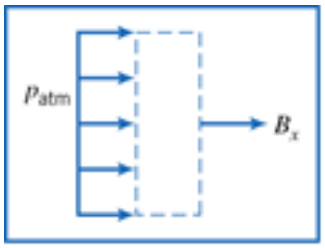
$$R_x = -15\left[\frac{m}{\sec}\right]\left|999\left[\frac{kg}{m^3}\right]\times 15\left[\frac{m}{\sec}\right]\times 0.01[m]\right|$$

$$R_x = -2.25[kN]$$

{R_x is the force acting on the control volume and it acts opposite to positive direction}

Force acting on the support $K_x = -R_x = 2.25\,[kN]$

II. Use C∀ II



No body force acts on this control volume in x-direction. Momentum equation in x-direction becomes

$$F_{S_x} = \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

$$F_{S_x} = p_A A + R_x = \int_{A_1} u\rho\vec{V}\cdot d\vec{A} = \int_{A_1} u\left\{-\left|\rho V_1 dA\right|\right\} = -2.25\left[kN\right]$$

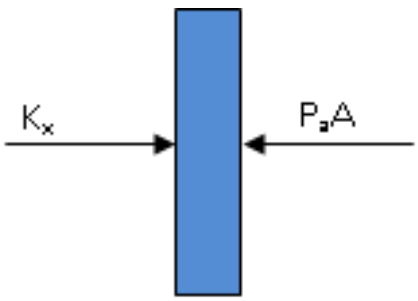
$$p_A A + R_x = -2.25\left[kN\right]$$

$$R_x = -p_A A - 2.25\left[kN\right]$$

and

$$K_x = -R_x = p_A A + 2.25\left[kN\right]$$

To determine the net force on the plate, we need to take into account pressure (atmospheric) force on the right face of the plate.

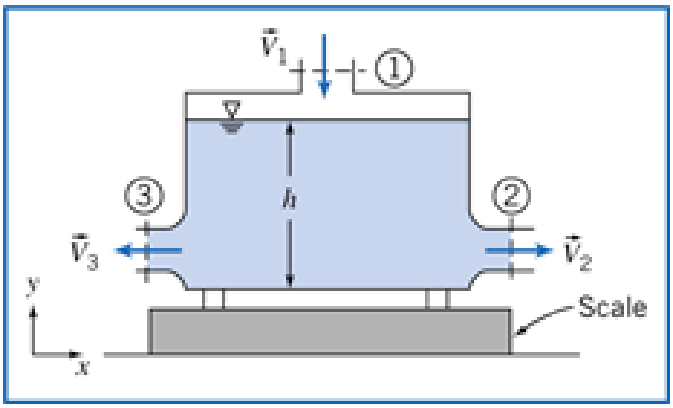


$$F_{net} = K_x - p_A A$$

$$F_{net} = p_A A + 2.25 - p_A A$$

$$F_{net} = 2.25\left[kN\right]$$

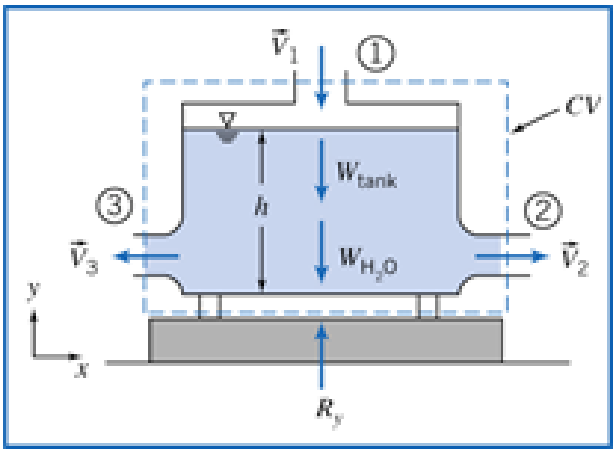
Example: A metal container, which has a height of **0.6 m** and an inside cross-sectional area of **0.1 m²**, is placed on a scale. Water flows into the tank at a velocity of **6 m/s** through an opening at the top with a cross-sectional area of **0.01 m²**, flows out the openings on the side walls with equal cross-sectional areas. Under steady flow conditions, the height of the water in the tank is **0.5 m.** The pressure is atmospheric across all openings, and the container weighs **50 N** when it is empty. If the frictional effects are negligible then determine the reading on the scale.



$A_T = 0.1\ m^2$
$V_1 = 6\ m/s$
$A_1 = 0.01\ m^2$
$A_2 = A_3$
$h = 0.5\ m$

$K_y = ?$

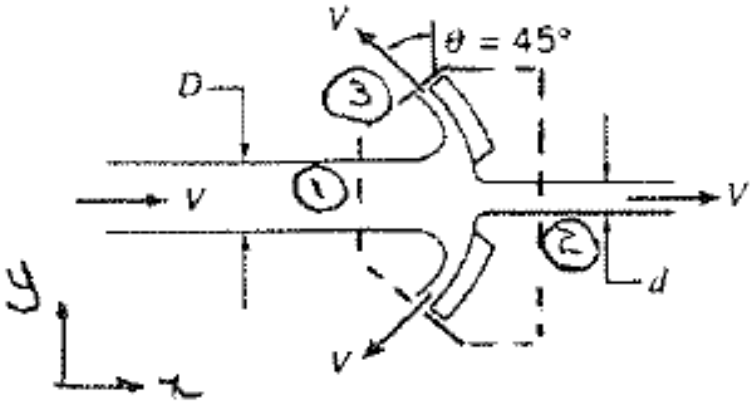
Solution



To be completed in class

Example: A shallow circular dish has a sharp-edged orifice at its center. A water jet of speed **V** strikes the dish concentrically. If the jet issuing from the orifice and from the surface of the dish also has speed **V**, **evaluate the external force needed to hold the dish in place for V = 5 m/s, D=100 mm and d=20 mm.**

Solution



To be completed in class